SIGNAL PROCESSING

Chapter 2 – Periodic signals & Fourier series

- **Trigonometric Fourier series (TFS)**

$$S_f(t) = \frac{a_0}{2} + \sum_{k=1}^{+\infty} a_k \cos(k\omega_0 t) + \sum_{k=1}^{+\infty} b_k \sin(k\omega_0 t)$$

$$with:$$

$$a_k = \frac{2}{T}\int_0^T s(t)\cos(k\omega_0 t)\,dt\,, b_k = \frac{2}{T}\int_0^T s(t)\sin(k\omega_0 t)\,dt$$

For a symmetric signal,

$$a_k = \frac{4}{T}\int_0^{T/2} s(t)\cos(k\omega_0 t)\,dt\,, b_k = 0$$

For an asymmetric signal,

$$b_k = \frac{4}{T}\int_0^{T/2} s(t)\sin(k\omega_0 t)\,dt\,, a_k = 0$$

- **Harmonic Fourier series (HFS)**

$$S_F(t) = \frac{A_0}{2} + \sum_{k=1}^{+\infty} A_k \cos(k\omega_0 t + \varphi_k)$$

$$with:$$

$$A_k = \sqrt{{a_k}^2 + {b_k}^2}\ ;\ A_0 = \frac{a_0}{2};\ \varphi_k = -\arctan\left(\frac{b_k}{a_k}\right)$$

- **Exponential Fourier series (EFS)**

$$S_F(t) = \sum_{k=\infty}^{+\infty} c_k\, e^{jk\omega_0 t}$$

$$with:$$

$$c_k = \frac{1}{T}\int_0^T s(t)\, e^{-jk\omega_0 t}\, dt$$

- **Relations between $\boldsymbol{a_k}$, $\boldsymbol{b_k}$, $\boldsymbol{A_k}$, $\boldsymbol{\varphi_k}$ and $\boldsymbol{c_k}$**

$$c_k = \frac{a_k - j\, b_k}{2}$$

$$|c_k| = \frac{A_k}{2} = \frac{\sqrt{{a_k}^2 + {b_k}^2}}{2}$$

$$\arg(c_k) = -\arctan\left(\frac{b_k}{a_k}\right) = \varphi_k$$

- **Power of a periodic signal**

$$P_\infty = \frac{1}{T}\int_0^T s^2(t)\, dt = \frac{{a_0}^2}{4} + \sum_{k=1}^{+\infty} \frac{{a_k}^2 + {b_k}^2}{2} = \frac{{A_0}^2}{2} + \sum_{k=1}^{+\infty} \frac{{A_k}^2}{2} = \sum_{k=1}^{+\infty} |c_k|^2$$

$$P_N = \frac{{a_0}^2}{4} + \sum_{k=1}^{N} \frac{{a_k}^2 + {b_k}^2}{2}$$